In the former case it is possible to avoid repeated numerical integration on every element and assemble the global stiffness matrix $\boldsymbol{S}$ efficiently by means of precomputed prototype integrals calculated on the reference domain K_a. The integrals present in the weak formulation of a concrete problem determine which constants have to be precomputed. For example, problem (2.20) with constant coefficients requires the $L^2(K_a)$-products of the first derivatives of the shape functions (master element stiffness integrals. MESI) and, if $a_0 \neq 0$, then also the $L^2(K_a)$-products of the shape functions themselves (master element mass integrals, MEMI), In one dimension these constants can be organized in the form of square matrices.

If we denote the maximum polynomial degree in the mesh by p_{max} and consider some set of shape functions $\varphi_1, \varphi_2, \ldots, \varphi_{p_{max}+1} \in P^{p_{max}}(K_a)$, the master element stiffness matrix $\boldsymbol{S}_{K_a}$ of problem (2.20) has the form

$$\boldsymbol{S}_{K_a} = \{\hat{s}_{ij}\}_{i,j=1}^{p_{max}+1} = \left\{ \int_{K_a} \varphi_i'(\xi)\varphi_j'(\xi)\,\mathrm{d}\xi \right\}_{i,j=1}^{p_{max}+1}. \tag{2.68}$$

The master element mass matrix $\boldsymbol{M}_{K_a}$ is defined as

$$\boldsymbol{M}_{K_a} = \{\hat{m}_{ij}\}_{i,j=1}^{p_{max}+1} = \left\{ \int_{K_a} \varphi_i(\xi)\varphi_j(\xi)\,\mathrm{d}\xi \right\}_{i,j=1}^{p_{max}+1}. \tag{2.69}$$

The only information about the reference map x_{K_m} that is needed on every element $K_m \in \mathcal{T}_{h,p}$ in the assembling algorithm is its Jacobian. Therefore, for each element K_m we introduce one more constant, `Elem[m].jac :=` $|J_{K_m}|$. The assembling procedure for model problem (2.20) with homogeneous Dirichlet boundary conditions can be written as follows.

Algorithm 2.5 (Assembling algorithm)

```
//Calculate the dimension of the space V_{h,p}:
N := -1;
for m = 1,2,...,M do N := N + Elem[m].p;
//Calculate the master element stiffness integrals MESI:
//(Use sufficiently accurate Gaussian quadrature to obtain exact results)
for i = 1,2,...,MAXP+1 do {
  for j= 1,2,...,MAXP+1 do {
    MESI[i][j] := ∫_{-1}^{1} φ'_i(x)φ'_j(x) dx;
  }
}
//Calculate the master element mass integrals MEMI:
for i = 1,2,...,MAXP+1 do {
  for j= 1,2,...,MAXP+1 do {
    MEMI[i][j] := ∫_{-1}^{1} φ_i(x)φ_j(x) dx;
  }
}
//Calculate the value of Elem[m].jac for all elements K_m, m = 1,2,...,M:
for m = 1,2,...,M do Elem[m].jac := (x_m - x_{m-1})/2;
//Set the stiffness matrix S zero:
for i = 1,2,...,N do for j = 1,2,...,N do S[i][j] := 0;
//Set the right-hand side vector F zero:
for i = 1,2,...,N do F[i] := 0;
//Element loop:
for m = 1,2,...,M do {
```