smooth functions that vanish on the boundary of Ω. The result also holds for every space $C^m(\Omega)$, $m \geq 0$, of m-times continuously differentiable functions (including the space $C(\Omega) = C^0(\Omega)$ of continuous functions).

Density argument A frequently used technique for proving various properties of functions in L^p-spaces, called density argument, works as follows:

- Let $v \in L^p(\Omega)$ be a function whose property (P) is to be shown.
- Take some sequence in $C^\infty(\Omega)$ converging to v in the $\|\cdot\|_p$-norm (the existence of such sequence is guaranteed by Lemma A.30).
- Prove that starting with some index n_0, the elements in the sequence have the property (P). This step usually is much easier for infinitely smooth functions than for the original function v.
- Show that also the limit of the sequence has the property (P).

This technique will be used, for example, to prove the Poincaré–Friedrichs' inequality in Paragraph A.4.5. Let us give an example of such sequence:

■ **EXAMPLE A.38 (Sequences of C^∞-functions converging to an L^p-function)**

1. In the interval $\Omega = (-1, 1)$ consider the function

$$v(x) = \begin{cases} 1 & x = 0, \\ 0 & \text{elsewhere in } \Omega. \end{cases}$$

This function belongs to the space $L^p(\Omega)$ for all $1 \leq p \leq \infty$. The sequence of $C^\infty(\Omega)$-functions $\{u_n\}_{n=1}^\infty$,

$$u_n = \left(1 - x^2\right)^n$$

converges to v in the p-norm for all $1 \leq p < \infty$. The functions $u_1, u_{10}, u_{100}, u_{1000}$, and u_{10000} are shown in Figure A.25.

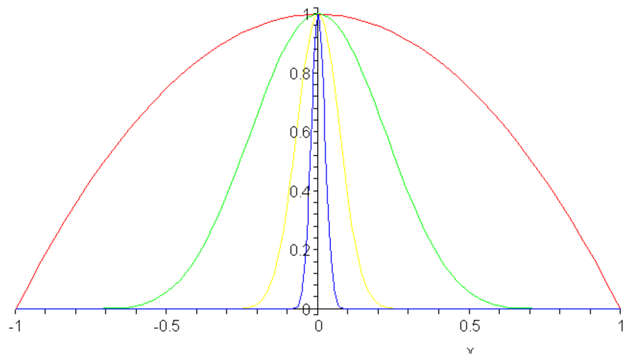

Figure A.25 Example of a sequence converging out of $C(-1, 1)$.

Since the function v is equivalent to the zero function in the Lebesgue sense, one can say that the sequence $\{u_n\}_{n=1}^\infty$ converges to zero in all spaces $L^p(\Omega)$ for all $1 \leq p < \infty$. The sequence does not converge in the L^∞-norm.