The basic assumption of the simple beam theory is that the normal deflection u is very small compared to the length of the beam, so that every pair of adjacent cross-sections A_1 and A_2, which are perpendicular to the axis of the beam in the original configuration, remain planar and perpendicular to the beam axis during the deformation.

The deflection of the beam can be described as the vertical displacement of the centroidal surface that corresponds to $z = 0$ in the initial configuration. In the situation shown in Figure 6.1, the deflection curve must be a circular arc (due to the homogeneity of the material, every cross section is subjected to the same stress and strain). By R we denote the radius of the deformed beam axis. Figure 6.2 shows the deformation of two small parallel length segments of originally identical lengths L, one lying on the beam axis and the other lying in the distance y from the axis.

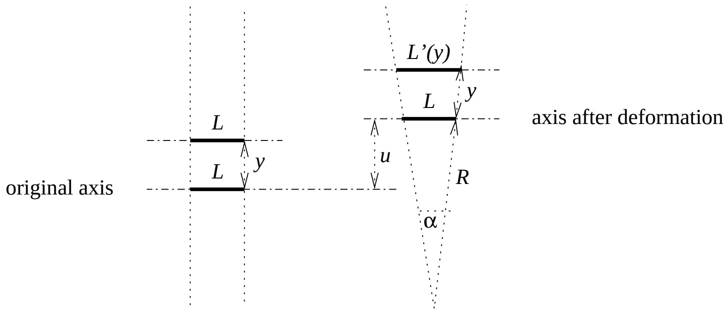

Figure 6.2 Strain induced within the deformed beam.

For a large radius R and small angle α we can write

$$L = R \sin \alpha, \quad L'(y) = (R + y) \sin \alpha,$$

and thus the axial strain $\epsilon(y)$, which is defined as the ratio of the length increment $L'(y) - L$ and the original length L, has the form

$$\epsilon(y) = \frac{L'(y) - L}{L} = \frac{y}{R}.$$

The response to the strain $\epsilon(y)$ is a stress $\sigma(y)$ which, according to the above assumptions, is one-dimensional in the direction of the x-axis. Hooke's law (6.1) yields

$$\sigma(y) = E\epsilon(y) = E\frac{y}{R}. \tag{6.2}$$

It follows from here that the centroidal plane $y = 0$ remains unstressed during the bending, i.e., that material particles on it are not strained in the axial direction. This plane is therefore called the neutral surface of the beam.

The moment resultant of the bending stress $\sigma(y)$ on every beam cross-section A must be equal to the external moment M,

$$M = \int_A y\sigma(y) \mathrm{d}A.$$