6.1 BENDING OF ELASTIC BEAMS

There are two basic one-dimensional models for the bending of elastic beams: The Euler–Bernoulli model consisting of one fourth-order PDE, and the Timoshenko model based on a pair of coupled second-order equations.

The Timoshenko model is simpler to solve in the sense that standard H^1-conforming elements can be used for its discretization, and it is known to better capture the purely three-dimensional behavior of the structure (such as large deformations). On the other hand, the higher-order elements used to discretize the Euler–Bernoulli case yield significantly better convergence rates. In this text we focus on the Euler–Bernoulli model in order to show the application of the Hermite and Argyris elements. The Timoshenko model is discussed quite frequently in monographs and textbooks (see, e.g., [95] and the references therein).

6.1.1 Euler–Bernoulli model

This paragraph requires the knowledge of some elementary topics in continuum mechanics that can be found, e.g., in [20, 95] or [124]. The one-dimensional Hooke's law has the form

$$\sigma = E\epsilon, \tag{6.1}$$

where σ is the stress induced by the strain ϵ, and E is the modulus of elasticity of the material.

Consider a prismatic beam of a homogeneous isotropic Hookean material with a rectangular cross section, whose longitudinal axis coincides with the x-axis of the given Cartesian system of coordinates. The position of the centroids of the end-faces is fixed, and a pair of bending moments acting on the ends of the beam is illustrated in Figure 6.1. (The downward-pointing z-axis is a convention used to make the signs of both the transversal load f and deflection u relative to the direction of gravity.)

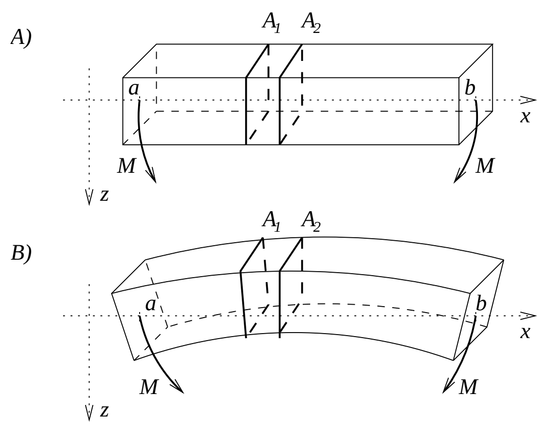

Figure 6.1 Bending of a prismatic beam; (A) initial configuration, (B) deformed state under moment M acting on the ends of the beam.