PREFACE

Rien ne sert de courir,
il faut partir à point.
Jean de la Fontaine

Many physical processes in nature, whose correct understanding, prediction, and control are important to people, are described by equations that involve physical quantities together with their spatial and temporal rates of change *(partial derivatives)*. Among such processes are the weather, flow of liquids, deformation of solid bodies, heat transfer, chemical reactions, electromagnetics, and many others. Equations involving partial derivatives are called *partial differential equations (PDEs)*. The solutions to these equations are functions, as opposed to standard algebraic equations whose solutions are numbers. For most PDEs we are not able to find their exact solutions, and sometimes we do not even know whether a unique solution exists. For these reasons, in most cases the only way to solve PDEs arising in concrete engineering and scientific problems is to approximate their solutions numerically. Numerical methods for PDEs constitute an indivisible part of modern engineering and science.

The most general and efficient tool for the numerical solution of PDEs is the *Finite element method (FEM)*, which is based on the spatial subdivision of the physical domain into *finite elements* (often triangles or quadrilaterals in 2D and tetrahedra, bricks, or prisms in 3D), where the solution is approximated via a finite set of polynomial *shape functions*. In this way the original problem is transformed into a *discrete problem* for a finite number of unknown coefficients. It is worth mentioning that rather simple shape functions, such as affine or quadratic polynomials, have been used most frequently in the past due to their relatively low implementation cost. Nowadays, higher-order elements are becoming increasingly popular due to their excellent approximation properties and capability to reduce the size of finite element computations significantly.

The higher-order finite element methods, however, require a better knowledge of the underlying mathematics. In particular, the understanding of linear algebra and elementary functional analysis is necessary. In this book we follow the modern trend of building engineering finite element methods upon a solid mathematical foundation, which can be traced in several other recent finite element textbooks, as, e.g., [18] (membrane, beam and plate models), [29] (finite element analysis of shells), or [83] (edge elements for Maxwell's equations).

The contents at a glance

This book is aimed at graduate and Ph.D. students of all disciplines of computational engineering and science. It provides an introduction into the modern theory of partial differential equations, finite element methods, and their applications. The logical beginning of the text lies in Appendix A, which is a course in linear algebra and elementary functional analysis. This chapter is readable with minimum prerequisites and it contains many illustrative examples. Readers who trust their skills in function spaces and linear operators may skip Appendix A, but it will facilitate the study of PDEs and finite element methods to all others significantly.

The core Chapters 1–4 provide an introduction to the theory of PDEs and finite element methods. Chapter 5 is devoted to the numerical solution of ordinary differential equations (ODEs) which arise in the semidiscretization of time-dependent PDEs by the most frequently used *Method of lines (MOL)*. Emphasis is given to higher-order implicit one-step methods. Chapter 6 deals with Hermite and Argyris elements with application to fourth-order problems rooted in the bending of elastic beams and plates. Since the fourth-order problems are less standard than second-order equations, their physical background and derivation are discussed in more detail. Chapter 7 is a newcomer's introduction into computational electromagnetics. Explained are basic laws governing electromagnetics in both their integral and differential forms, material properties, constitutive relations, and interface conditions. Discussed are potentials and problems formulated in terms of potentials, and the time-domain and time-harmonic Maxwell's equations. The concept of Nédélec's *edge elements* for the Maxwell's equations is explained.

Appendix B deals with selected algorithmic and programming issues. We present a universal sparse matrix interface sMatrix which makes it possible to connect multiple sparse matrix solver packages simultaneously to a finite element solver. We mention the advantages of separating the finite element technology from the physics represented by concrete PDEs. Such approach is used in the implementation of a high-performance modular finite element system HERMES. This software is briefly described and applied to several challenging engineering problems formulated in terms of second-order elliptic PDEs and time-harmonic Maxwell's equations. Advantages of higher-order elements are demonstrated.

After studying this introductory text, the reader should be ready to read articles and monographs on advanced topics including a-posteriori error estimation and automatic adaptivity, mixed finite element formulations and saddle point problems, spectral finite element methods, finite element multigrid methods, hierarchic higher-order finite element methods (hp-FEM), and others (see, e.g., [9, 23, 69, 105] and [111]). Additional test and homework problems, along with an errata, will be maintained on my home page.

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El Paso, Texas,
August, 2005